# Group Project 4
# Hyperbolic Functions
# Due Monday, Feb. 22

*****Read this first!** – You will need to email your professor (me) to get the specific integrals and derivatives to use in your report. Be sure to do this early enough to get this data and complete your report. (e,g. if you email me at 11:00 PM, I probably will not get you the data until the following day.)

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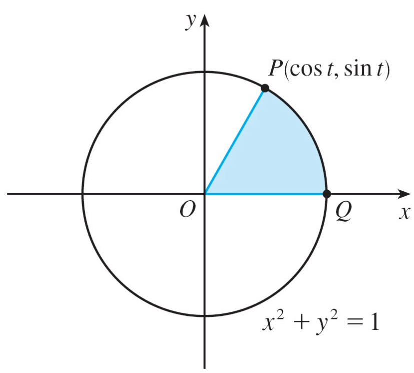
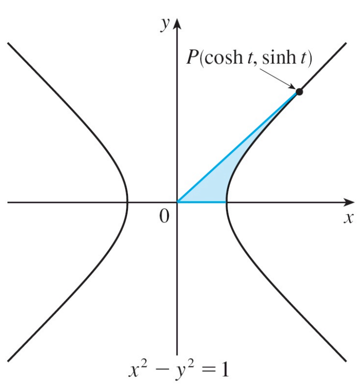
**Project Assignment:** The trigonometric function sine and cosine are often used in calculus. They are often defined in terms of the unit circle $x^2 + y^2 = 1$. For a given angle $t$ with the $x$ axis located at the origin we can locate a point $P$ on the circle. The coordinates of this point are $(\cos t, \sin t)$.




Similar functions can be defined using the unit hyperbola $x^2 - y^2 = 1$ (note the change of sign). For a given angle $t$ with the $x$ axis located at the origin we can locate a point $P$ on the hyperbola. The coordinates of this point are hyperbolic sine and cosine, $(\cosh t, \sinh t)$ (pronounced "cosh" and "singh").



Unlike the trigonometric functions, the hyperbolic sine and cosine have formulas that can be written in terms of other basic functions. If fact,

$$\sinh t = \frac{e^t - e^{-t}}{2} \text{ and } \cosh t = \frac{e^t + e^{-t}}{2}.$$

In this project you will investigate properties of these hyperbolic functions.

1. Use the formulas for the hyperbolic functions to verify that the point where $x = \cosh t$ and $y = \sinh t$ actually lies on the hyperbola $x^2 - y^2 = 1$. Note that this produces the identity $\cosh^2 t - \sinh^2 t = 1$. Draw a parallel to a similar trigonometric identity.

2. Again using the formulas, verify that hyperbolic sine is an odd function and that hyperbolic cosine is an even function. As before draw parallels to similar identities for trigonometric functions.

3. Graph both of these hyperbolic functions. Include your graphs in your report.

4. Define hyperbolic tangent, cotangent, secant and cosecant in a manner similar to the trigonometric functions. Include formulas for each of these in terms of the hyperbolic sine and cosine as well as formulas in terms of exponentials.

5. Find the derivatives for hyperbolic sine and cosine. Write your results in terms of other hyperbolic functions. Compare and contrast your results with similar results for trigonometric functions.

6. As with trigonometric functions we often use inverse functions to "undo" them. The inverse of hyperbolic sine is $\sinh^{-1}(t)$ and the inverse of the hyperbolic cosine is $\cosh^{-1}(t)$. There are formulas for these functions, but it turns out that what we most commonly use are formulas for their derivatives. Use implicit differentiation to show that

$$\frac{d}{dt}(\sinh^{-1} t) = \frac{1}{\sqrt{1+t^2}} \text{ and} \frac{d}{dt}(\cosh^{-1} t) = \frac{1}{\sqrt{t^2-1}}.$$

7. Use these results and substitutions to evaluate the integrals and derivatives assigned to you by your professor.

Your group should print your report and turn it in by the due date. Please be sure to fill out and return a project assessment form with your report.